Developing Teacher Understanding of Early Algebraic Concepts Using Lesson Study

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This paper reports on the use of lesson study as a professional development tool. In particular the paper focuses on the way in which the teachers increased their understanding of how tasks, classroom activity and teacher actions scaffolded student learning of early algebraic reasoning of equivalence and the commutative principle. Teacher voice is used to illustrate how lesson study cycles caused the teachers to reflect and review their own understandings of early algebraic concepts and how their students considered the concepts.

Introduction

Algebra is often provided as a reason for both the difficulties individuals encounter learning and making sense of mathematics, and the disaffection many people hold towards it. Given the position algebra holds in the educational and economical future of all individuals, Knuth and his colleagues (Knuth, Stephens, McNeil, & Alibabi, 2006) describe a growing consensus between researchers and educators that algebra be introduced at a much younger age with a focus on the integration of teaching and learning arithmetic and algebra in classrooms. This emphasis is confirmed in policy documents which describe a unified curricula strand (e.g., Department for Education and Employment, 1999; National Council of Teachers of Mathematics 2000). Teachers, within this changing context are required to find ways to make algebra accessible to all their students, through the use of rich learning tasks in environments which provide all students opportunities to learn algebra with rich, conceptual understanding (Chazan, 1996). The focus of this paper is on how a group of teachers used lesson study to explore how some designed tasks could be used to better support student development of key early algebraic concepts.

Teachers have a key role in reforming classroom practice and activities which integrate arithmetic and algebra. But, we know that for many this poses considerable challenges; they may not have understandings of how to make links between arithmetic and algebra, nor may they have had experience constructing and using rich connected types of integrated (arithmetic/algebra) problems. As Blanton and Kaput (2003) suggest, they may not have developed their algebra 'ears and eyes' when working with the patterns and relationship in number which promote rich connected conceptual understandings. Blanton and Kaput suggest a remedy for this situation could be a form of professional development in which opportunities are structured so teachers identify numerical patterns and relationships which connect to early algebraic reasoning. In this paper lesson study was used as a form of professional development to facilitate a group of teachers enhanced algebraic 'eyes and ears'. The aim of this paper is to explore how professional development in the form of lesson study supported the teachers to 'notice' opportunities for developing student's early algebraic reasoning. The questions asked in this study were: How did the use of lesson study support teachers to comprehend how their students understood the key concepts of equivalence and the commutative principle, and; how did the repeated cycles in lesson study facilitate the teachers to identify the challenges involved in students' constructing conceptual understandings of equivalence and the commutative principle.

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Lesson Study

Lesson study, developed in Japan, is one form of professional development which aims to increase teachers' knowledge about mathematics, knowledge about ways of teaching mathematics, and knowledge about the ways in which learners engage with and make sense of mathematics (Fernandez & Yoshida, 2004). In the lesson study format a group of teachers collaboratively plan a lesson (termed the 'study lesson') over a series of meetings. A cycle is developed. The 'study lesson' is taught by a team member, observed by other members with particular focus on student responses. Then in subsequent meetings the observed 'study lesson' is discussed, analyzed, reconstructed in line with student responses, then re-taught to a different group of students. This cycle may be repeated or different lessons developed. In this paper lesson study was used as a form of continuing professional development (CPD), which focused on enhancing specific aspects of teacher knowledge but with a particular emphasis on student learning.

Developing Early Algebraic Reasoning

Students constructing rich conceptual understandings of algebraic reasoning takes a long time and requires that their attention is placed on the inter-related connections across all other types of mathematics, and particularly arithmetic (Blanton & Kaput, 2005). The students' intuitive knowledge of patterns and numerical reasoning are used to provide a foundation for transition to early algebraic thinking (Carpenter, Franke, & Levi 2003). Carpenter and his colleagues explain that for students to justify, and generalise their mathematical reasoning about the properties of numbers they also need to be provided with opportunities to make conjectures in the classroom environment. Research studies investigating young children's development of early algebraic reasoning covers a wide field including those which focus on classroom practices which scaffold student justification and generalisations. However, in this paper to explore how lesson study supported a group of teachers to develop understanding of how their students constructed early algebraic reasoning the focus is narrowed to two areas of early algebra, equivalence (equality) and the commutative principle. The next section makes a brief examination of the literature related to equivalence, the commutative property and lesson study as a professional development process.

Equivalence

Developing understanding of equality is a concept fundamental to algebraic reasoning. Kieran (1981) in her seminal studies illustrated that many elementary school students have an inadequate understanding of the equal sign. Other studies (e.g., Carpenter et al., 2003; Knuth et al., 2006) concur. The difficulties these students encounter are caused because they view the equal sign as an indicator of an operator rather than a symbol of a mathematically equivalent operation. This limits the strategies they have available to solve equivalence problems and in later years symbolic equations (Knuth et al.). To address this problem, teachers need to be aware of how many students view the equal sign and construct and use activities in the classroom which expand student understandings of the equal sign and ensure that the misconceptions are identified and addressed. A range of successful classroom interventions (e.g., Carpenter, et al., 2005; Molina, Castro, & Castro, 2009) which enriched student understanding of equivalence have included non-standard representations such as true and false number sentences and balance scale representations.

Commutative Principle

Opportunities to explore the properties of numbers and operations provide a rich platform for developing algebraic reasoning. However, many exploratory studies (e.g., Anthony & Walshaw 2002, Warren 2001) illustrate that elementary students often have limited classroom experiences in exploring the properties of numbers and operations. As a result the students lack understanding of the operational laws and are unable to construct correct generalisations of the commutativity principle. Anthony and Walshaw illustrated that many students generalised the commutative nature of addition and multiplication, but over-generalised the relationship to include subtraction and division. They showed that while some students could explain the commutative property they could not construct generalised statements nor use materials to model their conjectures. However, studies by Blanton and Kaput (2003) and Carpenter and his colleagues (2003) provided clear evidence that when young children are provided with opportunities in the classroom they learn to construct and justify generalisations about the fundamental structure and properties of numbers. Importantly, these studies demonstrated that when classroom activity targeted students' numerical reasoning they explored, constructed and validated conjectures using appropriate generalisations and justifications

Theoretical Framework

The theoretical framing of this paper is based within a socio-cultural perspective. In this view the processes of teaching and learning hold a reciprocal relationship. The teaching is integrally connected to student learning as manifested through the changing competencies and disposition of the students. In turn, the teachers' professional development is interrelated and identified through evidence of their actions in the classroom, and changes in their professional competencies and attitudes.

Methodology

This paper reports on episodes drawn from a larger study which involved a year-long continuing professional development classroom-based intervention. The participants included two separate groups of elementary teachers (one group from England the other from the Channel Islands). The sample was an opportunistic one of teachers who wanted to extend understandings of ways to facilitate young students' development of early algebraic reasoning. This paper specifically reports on one section of the larger study. In this section the teachers engaged in lesson study for the first time although the Beaumont School teachers had engaged in a paired collaborative observation approach the previous school year, teachers at Hillview School had no experience using collaborative approaches to planning or teaching. The schools were a mixture of rural and suburban contexts and the students came from a range of socio-economic and ethnic backgrounds. The teachers had varying levels of experience.

In the lesson study process used in this study each group of teachers worked as a professional learning community within their own school. Over-arching aims relevant to each school were established immediately. These collaboratively agreed goals broadly established that the teachers wanted to develop creative and flexible problem solvers. Then all members of the research team (the teacher groups and researchers) planned an area of focus for the study lessons. The foci corresponded to mathematical concepts their students had difficulties with or those which the teachers felt less confident about teaching. Through collaborative activity 'study lessons' were planned and taught in one classroom and observed by the research group In-depth analysis and discussion followed observations of the study lesson and subsequent iterations as it was re-planned, re-taught and re-observed in different classrooms as part of the lesson study cycle.

The lesson study cycles in the two settings differed. At Hillview the teachers wanted to address how their students over-generalised the commutative principle to include subtraction and division. A lesson study cycle was devised which included lessons designed to facilitate student understanding and justification of the commutative property with a focus on the use of representations to model conjectures and justify reasoning. The students were given the following statement made by a student in an earlier lesson: 'If you have two numbers and you are adding them it does not matter which number you add first the answer will still be the same.' The students worked in small groups of four and explored with equipment whether the statement held when applied to the different operations. They were required to model their reasoning with equipment, as well as represent it verbally, symbolically and solve problems which involved multiple operations. At Beaumont the study lesson cycles aimed to develop students' skill at solving multi-step word problems and part of the focus was placed on the equal sign. The students were asked to make a specific number using a number sentence which was then represented as equivalent to another number sentence (for example 45 = 20 x 2 + 5 = 20 + 25 = 45 – 0) and included some incorrect multi-step equations.

Data gathering included detailed field notes, video and audio records of the planning meetings and classroom lessons and artefacts. The video and audio recordings were wholly transcribed and through an iterative process using a grounded approach, patterns, and themes were identified. The on-going and retrospective data analysis supported the development and construction of case studies of the two study groups. Evidence was triangulated using classroom observations, artefacts and analytical discussion.

Findings

The first section outlines how the use of the lesson study process facilitated the teachers to notice key aspects of early algebraic reasoning and included both planned opportunities for student learning and spontaneous opportunities which arose as tasks were enacted in classrooms.

Developing Understanding of Student Approaches to the Tasks

Discussion and analysis of student responses in the study group illustrated that opportunities to closely observe student responses during the lesson provided a foundation for the teachers to build understandings of how students approach tasks which challenge their understandings of the commutative principle. The teachers expressed surprise that many students began with the use of counter-examples to show that the commutative property did not apply to subtraction. For example Ellen commented:

> Iris and her partner were looking at subtraction without even being prompted to do it because they said straight away "it doesn't work for subtraction but it is working for addition.

Similarly, in further discussion another teacher noted that the students initiated their investigation with the development of a counter-example:

> They were doing it with the subtraction. They did four minus one equals three and one minus four and Lauren said '"so that is subtraction done then, that doesn't work" and she did it for one, if it doesn't work, it doesn't work whereas she then said "actually five times three and three times five works hmm". Then they did something with twos and then she said "does it only work with twos though". So then they tried with a different number. That was Lauren who said that so she had got the

idea that if with one it didn't work, she just discarded that straight away and went straight onto the next one.

Through their observations in the lesson study they had observed how the students intuitively realised that a counter-example disproved the conjecture. However, the students would explore further with other numbers if the conjecture appeared to be correct; in that situation they were not satisfied that one example proved a conjecture.

Developing Understanding of the Role of Materials in Sense-making

Lesson study provided opportunities to develop teacher knowledge of how students could justify their conjectures using materials. For example, in a study group discussion it was evident that a teacher lacked understanding of how the children could justify their conjectures through use of an array. During the observed lesson two groups of students justified an explanation that multiplication was commutative through use of an array. The teacher did not use their explanation to extend the other students' reasoning. Then when another student had difficulties articulating the same concept the teacher stopped her explanation. In the post lesson discussion the researcher stated what the student was explaining:

Researcher: What Andrea was trying to say but she couldn't quite articulate it was if you just kept making it longer it could be any number because you could just keep adding on and it is still the same amount multiplied by the same amount.

Monica: I was conscious of the time, the bell was going to go and I wasn't sure of what she was trying to say from where I was standing.

The teacher's response illustrates that she did not understand how an array supported the explanation nor could she build on and extend the students' explanations of the commutative property. This was reiterated during further analysis in the follow-up discussion. As the other teachers explained and analyzed the student responses the teacher clarified her own actions:

Monica: I didn't know what he was saying about the two numbers.

Melissa: He was moving the rows.

Ellen: Yes, he was saying to turn them around.

Melissa: He moved the rows, he said look you don't have to…

The teacher explains from her own point of confusion her response:

Monica: I thought actually that might have confused everybody else.

Ellen: But he knew what he meant so he could explain it the other way.

At this point Monica acknowledged that because she was confused by what the representation showed she assumed many students would also be.

The teachers also became aware of how important it is that students have access to equipment to scaffold their understanding. After observing that a group of students encountered difficulties investigating whether division was commutative Ellen commented:

When they had the pegs in front of them then they could argue it but they couldn't argue it just on paper. They needed to be able to see the five pegs and they can't divide them amongst the ten people and quite a few groups were like that.

The teachers also observed the way in which the students used equipment to link to real-life situations to model their reasoning:

> I think it was Iris and Andrea, they were talking about the objects and they suddenly became sweets, “if we have got three sweets we can't divide them between seven people” so they were then jumping ahead and moving that relationship on, that was good. I think it was the resources that prompted that.

During further discussion the teachers illustrated how they now understood how physical representations supported students to work at higher levels of generality:

> Monica: I think even with John if he hadn't seen it on the grid [referring to an array constructed on a pegboard] he probably wouldn't have got it as quickly as he did.
>
> Melissa: Because he had really got it in his head, hadn't he? Because he wasn't even really sure if six times four what it equaled, he just knew that it was the same.
>
> Monica: Originally he was convinced that it didn't work so it was only after Sridatta disagreed and showed him it on the grid.
>
> Melissa: The fascinating part is he didn't even work out what the answer was. It didn't matter, it was irrelevant [indicates turning array with hands].

However, the teachers were surprised at the difficulties students had using equipment to model and justify conjectures. During the first lesson cycle their attention was drawn to how the procedural use of symbols dominated how the students responded to tasks:

> Melissa: The thing is them trying to use them as symbols and they got fixated on the idea, like that group over, they even had the scissors as an equals sign
>
> Ellen: And using the blocks to try and create the numbers... I think making it explicit that the objects are representative of a proper number and that they are not to then start creating equations out of them. We don't want to see them as numbers but as objects.

Again the next lesson cycle drew their attention to the student attempts to use materials as symbols:

> Melissa: I think that’s the same thing again, they wanted to start putting in the signs and symbols...so they had three colours then a white peg, then one peg and then a white peg and then four pegs and she said “we’ve put that peg to mean add” so they were doing the same thing. It’s like they need to have the symbols there rather than just having like their array as a justification.

Developing Knowledge of How Understanding of the Commutative and Equal Sign is Constructed Over Time and through Specific Teacher Actions

The study group discussion provided many opportunities for the teachers to reflect on what happened in the study lessons and identify missed teaching opportunities. For example in one section of a lesson the discussion focused on examining the structure of multiplication operations and the teacher shifted the children from the general to the specific by guiding them to solve the equations to show the answers were the same. In the analytical discussion with colleagues she recognized that by directing the students towards answers rather than the general structure of multiplication some students became focused on specific equations rather than generalized understandings of the commutative nature of multiplication:

> Monica: I shot myself in the foot because I did that because I knew that some of them hadn't got it so I wanted to show them that actually you know you could tell if you worked them out separately. You could ascertain they had the same answer but then it kind of made other people get stuck at that stage.

In the continuing conversation she saw how her actions caused many students to use procedural rather than conceptual understandings.

The lesson cycles also provided a foundation for the teachers to recognise the need to press students beyond specific examples to generalised reasoning. After a second lesson cycle a teacher observed:

> Melissa: It is almost as though because they had chosen numbers that were simple enough that they knew that two times four made eight so they weren't looking at them as an array, they were looking at them as if that is the numbers that we are dealing with.

This statement led to further discussion in the group, of actions they could use to scaffold the students towards more generalised reasoning.

The post-lesson discussions also provided evidence of the teachers' growing ability to notice student misconceptions of early algebraic reasoning beyond that of the lesson foci. One example occurred when the teachers discussed the difficulties the students had in representing the commutative principle as a number sentence (for example 6 + 5 = 5 + 6).

> Ellen: They seem to find it really hard to write one continuous number sentence.

In response Monica drew the groups' attention to the on-going difficulties the students had with the equal sign as a concept of equivalence:

> They are still not understanding the proper meaning of the equal sign or perhaps they are but when it comes to applying it in a context then they're not.

In the teacher discussions evidence was provided that they became aware that constructing understanding of equivalence is a lengthy and difficult process which requires a press from the teacher and a lot of student discussion and exploration:

> Zara: We still had to keep coming back to that, that the two sides of the equation had to balance. How much time we have done that, and even given that they had done that in the first part of the lesson. They don't seem to see that as the same.

Within this discussion on-going analysis of the observation and how the activities caused students to think about equivalence led to further analysis and reflection from another teacher.

> Rebecca: I think maybe because we historically present children with a lot of things with the answer just being one box that sort of one where they had to look maybe provoked that thinking a little bit more. You know at the beginning where they said something, something equals and then the next child does equals, I don't know, when I look at it now I think it is a fantastic activity and a fantastic assessment...but maybe they are just seeing and the next one, and the next one, and now it's my turn and they don't actually see the equal sign whereas this question here and that one here in particular really made them think about the idea of balance.

In this statement the teacher has voiced her growing awareness of why students develop misconceptions around the equal sign and the importance of considering how teacher actions coupled with rich tasks structure how students make links between arithmetical and algebraic reasoning.

Discussion and Conclusions

The use of teacher voice in the study group discussions sheds light on the many learning opportunities the teachers encountered as they observed and listened to student activity during the lesson study. Clearly they observed the pivotal role the teacher had, in the study lessons, in making links between the early arithmetical and algebraic reasoning and pressing the students towards situations of generality. Their algebra 'eyes and ears' (Blanton & Kaput, 2003) became more attuned to recognizing common misconceptions as the teachers worked together in the lesson cycles. They also developed cognizance of the need to better match their actions to the classroom discussions and activity.

Of importance in this study was the teachers' recognition of the many challenges they face in developing students' rich connected learning about equivalence and the commutative principle. As previous researchers (e.g., Carpenter et al., 2003; Blanton & Kaput, 2005) note, students have common misconceptions but for the teachers recognition of these caused reflective and analytic discussions. The findings of this paper suggest that lesson study has considerable promise as a learning tool for teachers to support them reforming their practices to integrate arithmetic and algebra.

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